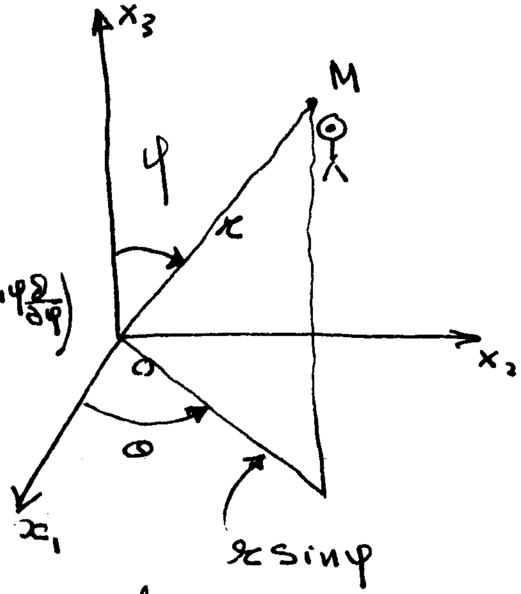
Sommerfeld Radiation Condition

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p = 0$$

In spherical Coordinates

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial}{\partial\varphi}\right) + \frac{1}{r^2\sin^2\varphi}\frac{\partial^2}{\partial\theta^2}$$



At large distance from the source, $r \to \infty$

$$\nabla^2 \sim \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + O(1/r^2)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p \sim \frac{1}{r}\left(\frac{\partial^2}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(rp) + O\left(\frac{1}{r^2}\right)$$

Or

$$\left(\frac{\partial^2}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(rp) = O(1/r)$$

$$\left(\frac{\partial}{\partial r} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial r} + \frac{\partial}{c\,\partial t}\right)(rp) = O(1/r)$$

Causality suggest that the wave must be propagating outward.

$$rp = f(r-ct) + \cancel{g(r+ct)} + O\left(\frac{1}{r}\right)$$

$$rp = f(r-ct) + O(1/r)$$

$$p = \frac{f(r-ct)}{r} + o(1/r^2)$$

$$\left(\frac{\partial}{\partial r} + \frac{1}{c}\frac{\partial}{\partial t}\right) P = -\frac{f}{r^2} + o(1/r^2)$$

$$r\left(\frac{\partial}{\partial r} + \frac{1}{c}\frac{\partial}{\partial t}\right) P = o(1/r)$$

$$\lim_{r \to \infty} \left[r \left(\frac{\partial}{\partial r} + \frac{1}{c}\frac{\partial}{\partial t}\right) P \right] = 0.$$

For a constant frequency case,

$$\lim_{r \to \infty} \left[r \left(\frac{\partial}{\partial r} - i\,\frac{\omega}{c}\right) P \right] = 0$$